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SOME PROPERTIES OF THE UPPER SEMILATTICE OF COMPUTABLE FAMILIES OF COMPUTABLY ENUMERABLE SETS

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We look at specific features of the algebraic structure of an upper semilattice of computable families of computably enumerable sets in Ω. It is proved that ideals of minuend and finite families of Ω coincide. We deal with the question whether there exist atoms and coatoms in the factor semilattice of Ω with respect to an ideal of finite families. Also we point out a sufficient condition for computable families to be complemented.

Let $\mathcal{S}$ be some family of computably enumerable (c.e.) sets. A *numbering* of the family $\mathcal{S}$ is an arbitrary mapping ν of the natural series $\mathbb{N} = \{0, 1, \dots\}$ on $\mathcal{S}$. A numbering ν is said to be *computable* [1] if $G_\nu = \{\langle x, y\rangle : y \in \nu(x)\}$ is a c.e. set. A family of c.e. sets is *computable* if it possesses at least one computable numbering. We denote the set of all computable numberings of $\mathcal{S}$ by $\mathrm{Com}(\mathcal{S})$, and the class of all computable families by Ω.

The class Ω is union-closed, and so the poset $\langle \Omega; \subseteq\rangle$ forms an upper semilattice with a greatest element $\mathcal{E}$, the family of all c.e. sets, and a least element $\varnothing$. The given semilattice was introduced and first studied in [2]. In the present paper, we will look at some new characteristics of its algebraic structure.

We will adhere to the notation and terminology used in [1, 3]. For a partial function ψ, its domain and range are denoted by $\delta\psi$ and $\rho\psi$ respectively. Let φ_n be a unary partial computable

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