Magazine of Civil Engineering. 2019. 91(7). Pp. 129–144
Инженерно-строительный журнал. 2019. № 7(91). С. 129–144

Magazine of Civil Engineering

ISSN
2071-0305

journal homepage: http://engstroy.spbstu.ru/

DOI: 10.18720/MCE.91.12

Dynamic deformation of a beam at sudden structural transformation of foundation

V.I. Travush[a], V.A. Gordon[b], V.I. Kolchunov[c*], E.V. Leontiev[d]
[a] Russian academy of architecture and construction science, Moscow, Russia
[b] Orel State University named after I.S. Turgenev, Orel, Russia
[c] Southwest State University, Kursk, Russia
[d] Main State Expertise of Russia, Moscow, Russia
** E–mail: asiorel@mail.ru*

Keywords: beam, foundation, natural vibrations, forced vibrations, mode, frequency, accidental impact, structural transformation

Abstract. The article presents a methodic for analytical determining forces, displacements, modes and frequencies of natural flexural vibrations of a beam on elastic foundation. The beam consists of two sections: the first one supports on Winkler elastic foundation, and next one is free. Equations for flexural natural and forced vibrations were written in dimensionless variables and parameters and solved using the initial parameters method and Krylov functions. At the same time second and higher frequencies of natural vibrations of the beam were determined assuming unknown frequency is higher than "conventional" frequency which characterizes generalized stiffness of a system "beam–foundation". Using numerical analysis, authors showed dependencies between the first three dimensionless frequencies of natural vibrations of the beam and a generalized stiffness of the system "beam–foundation" when foundation suddenly partially failure under the beam. Investigation established that effect of a sudden structural transformation leads to five-time moment increasing in the system "beam–foundation" at sudden foundation failure under the second half of the beam.

1. Introduction

Number of investigations on defense of buildings and structures against progressive destruction increases permanently [1] and most of these works deal with load redistribution in structural systems when a constructive element is removed from a building frame [2–4]. Investigations on deformation features of structures in a system "structure–foundation" under accidental impacts caused by sudden damage of a foundation are practically absent [13, 24]. A negligible number of studies [5–12] describes dynamic performance of beams and piles partially supported by an elastic base or partially imbedded into such a base. At the same time, it is usually assumed that a free structural segment is initially designed as a quasi-static body disregarding the inertia force [7–12]. In this regard, a problem of dynamical effects appearing when damage of a part of the system "beam–foundation" (such a partial destruction, boundary condition changing, crack formation, layers separation, reinforcement rupture) occurs suddenly, is of theoretical and practical interest [2, 25–27]. In this paper, we formulate and solve a problem on determination of dynamical force increment in a beam supported by an elastic base of Winkler's type during forced transverse oscillations of the beam due to sudden partial damage of the base. The paper presents results on analytical determination of forces, modes, and frequencies of a beam supported by elastic foundation when sudden partial destruction of foundation does occur.

Travush, V.I., Gordon, V.A., Kolchunov, V.I., Leontiev, E.V. Dynamic deformation of a beam at sudden structural transformation of foundation. Magazine of Civil Engineering. 2019. 91(7). Pp. 129–144. DOI: 10.18720/MCE.91.12

Травуш В.И., Гордон В.А., Колчунов В.И., Леонтьев Е.В. Динамическое деформирование балки при внезапном структурном изменении упругого основания // Инженерно-строительный журнал. 2019. № 7(91). С. 129–144. DOI: 10.18720/MCE.91.12

2. Methods

The paper presents a formulation of the problem on determination of modes and frequencies of natural transverse oscillations for a beam of length L with flexural stiffness EI and distributed mass ρA and consisting of two segments (Figure 1): the first segment of length L_1 is supported by an elastic foundation of Winkler's type and the second one of length $L - L_1$ is free. Solutions of the vibration problem for these two segments are constructed separately. Integration constants for proper differential equation can be determined owing to the conjunction conditions between beam's segments along with the boundary conditions at beam's endpoints.

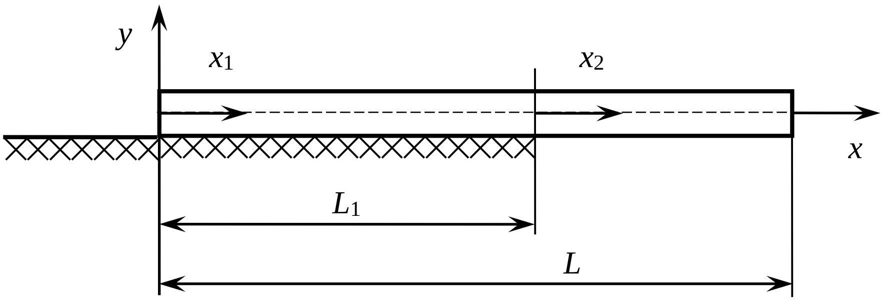


Figure 1. Beam partially supported by elastic foundation.

Transverse oscillations of the first segment $0 \le \xi_1 \le \nu$

Let us introduce dimensionless variables and parameters

$$\xi_i = \frac{x_i}{L}\ (i = 1, 2);\ \ w_i = \frac{v_i}{L};\ \ \tau = \omega_0 t;\ \ \nu = \frac{L_1}{L};\ \ \omega_0 = \sqrt{\frac{K}{\rho A}};\ \ \alpha = \sqrt[4]{\frac{KL^4}{4EI}};\ \ \bar{\omega} = \frac{\omega}{\omega_0},$$

where x_i – axial coordinate for i-th segment (i = 1, 2);

$v_i = v_i(x_i, t)$ is deflection field of i-th segment (i = 1, 2);

ν is relative length of the segment supported by foundation;

$K = kb$ is foundation stiffness;

k is modulus of subgrade reaction,

b is width of cross section;

α is generalized stiffness of the system «beam is foundation»;

ω_0 is "conventional" frequency parameter that takes dimension of frequency [s^{is1}];

t is physical time;

ω is frequency of natural oscillations.

The equation of natural oscillations for the first section takes the form [25, 26]

$$\frac{\partial^4 w_1}{\partial \xi_i^4} + 4\alpha^4 \left(w_1 + \frac{\partial^2 w_1}{\partial \tau^2} \right) = 0. \qquad (1)$$

Let us find the solution of the equation (1) assuming that oscillations are harmonic and using separation of variables:

$$w_1(\xi_1, \tau) = W_1(\xi_1) \sin \frac{\tilde{\omega}}{\bar{\omega}_0} \tau, \qquad (2)$$

where $\bar{\omega} = \dfrac{\omega}{\omega_e}$ is dimensionless frequency;

$\bar{\omega}_0 = \dfrac{\omega_0}{\omega_e}$ is dimensionless "conventional" frequency;

$\omega_e = \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}}$ is “reference” frequency. Further, we use the dimensionless «conventional» frequency $\bar{\omega}_0$, as a generalized stiffness of the system “beam–foundation” instead of parameter α which has a physical meaning. Reducing the base stiffness K by means of the relationships

$$\omega_0 = \sqrt{\frac{K}{\rho A}} \text{ and } \alpha = \sqrt[4]{\frac{KL^4}{4EI}},$$

and taking into account the reference frequency $\omega_Э$ we obtain

$$4\alpha^4 = \frac{\omega_0^2}{\frac{1}{L^4}\frac{EI}{\rho A}} = \left(\frac{\omega_0}{\omega_e}\right)^2 = \bar{\omega}_0^2.$$

Substituting (2) into (1), we obtain the equation for modes of natural oscillations

$$W_1^{\mathrm{IV}} + \left(\bar{\omega}_0^2 - \tilde{\omega}^2\right)W_1 = 0. \quad (3)$$

The structure of equation (3) yields the following three possible solutions:

1) if $\tilde{\omega} > \bar{\omega}_0$, then, writing the equation (3) in the form

$$W_1^{\mathrm{IV}} - \left(\tilde{\omega}^2 - \bar{\omega}_0^2\right)W_1 = 0 \quad (4)$$

and solving the last equation by Euler’s substitution

$$W_1 = Ae^{r\xi_1}, \quad (5)$$

we obtain the characteristic equation

$$r^4 - \left(\tilde{\omega}^2 - \bar{\omega}_0\right) = 0,$$

the roots of which are as follow:

$$r_{1,2} = \pm\beta_1; \quad r_{3,4} = \pm i\beta_1; \quad \beta_1 = \sqrt[4]{\tilde{\omega}^2 - \bar{\omega}_0^2}. \quad (6)$$

Then the solution of equation (4) takes the form

$$W_1(\xi_1) = W_{10}R_4(\beta_1\xi_1) + W_{10}'R_3(\beta_1\xi_1) + W_{10}''R_2(\beta_1\xi_1) + W_{10}'''R_1(\beta_1\xi_1), \quad (7)$$

where $W_{10}, W_{10}', W_{10}'', W_{10}'''$ are initial parameters;

$R_i = R_i(\beta_1\xi_1)$ is Krylov’s function;

$$R_1 = \frac{\mathrm{sh}\beta_1\xi_1 - \sin\beta_1\xi_1}{2\beta_1^3}; \quad R_2 = \frac{\mathrm{ch}\beta_1\xi_1 - \cos\beta_1\xi_1}{2\beta_1^2};$$

$$R_3 = \frac{\mathrm{sh}\beta_1\xi_1 + \sin\beta_1\xi_1}{2\beta_1}; \quad R_4 = \frac{\mathrm{ch}\beta_1\xi_1 + \cos\beta_1\xi_1}{2}; \quad R_4' = \beta_1^4 R_1.$$

The matrix equation for the state of arbitrary cross section ξ_1 in the first segment reads as

$$\bar{W}_1(\xi_1) = V_{11}(\beta_1\xi_1)\bar{W}_{10}, \quad (8)$$

where $\bar{W}_1(\xi_1) = \left\{W_1(\xi_1)\, W_1'(\xi_1)\, W_1''(\xi_1)\, W_1'''(\xi_1)\right\}^T$ is state vector of an arbitrary cross section ξ_1;

$\bar{W}_{10} = \left\{W_{10}\; W_{10}'\; W_{10}''\; W_{10}'''\right\}^T$ is initial parameters vector;

$$V_{11}(\xi_1)=\begin{pmatrix} R_4(\beta_1\xi_1) & R_3(\beta_1\xi_1) & R_2(\beta_1\xi_1) & R_1(\beta_1\xi_1) \\ \beta_1^4R_1(\beta_1\xi_1) & R_4(\beta_1\xi_1) & R_3(\beta_1\xi_1) & R_2(\beta_1\xi_1) \\ \beta_1^4R_2(\beta_1\xi_1) & \beta_1^4R_1(\beta_1\xi_1) & R_4(\beta_1\xi_1) & R_3(\beta_1\xi_1) \\ \beta_1^4R_3(\beta_1\xi_1) & \beta_1^4R_2(\beta_1\xi_1) & \beta_1^4R_1(\beta_1\xi_1) & R_4(\beta_1\xi_1) \end{pmatrix} \quad (9)$$

is a functional matrix describing the initial parameters influence on the state of cross section ξ_1 in the first segment.

2) if $\tilde{\omega} < \bar{\omega}_0$, then, substituting equation (5) into (3), we obtain the characteristic equation

$$r^4+\left(\bar{\omega}_0^2-\tilde{\omega}^2\right)=0$$

with the complex roots

$$r_{1\div 4}=(\pm i\pm 1)\beta_2;\quad \beta_2=\sqrt[4]{\frac{\bar{\omega}_0^2-\bar{\omega}^2}{4}} \quad (10)$$

and the solution of the equation (3) reads as

$$W_1=W_{10}K_4(\beta_2\xi_1)+W'_{10}K_3(\beta_2\xi_1)+W''_{10}K_2(\beta_2\xi_1)+W'''_{10}K_1(\beta_2\xi_1), \quad (11)$$

where $K_i=K_i(\beta_2\xi_1)$ are Krylov's functions that are of the form

$$K_1=\frac{\mathrm{sh}\beta_2\xi_1\mathrm{ch}\beta_2\xi_1-\cos\beta_2\xi_1\mathrm{sh}\beta_2\xi_1}{4\beta_2^3};\quad K_2=\frac{\sin\beta_2\xi_1-\mathrm{sh}\beta_2\xi_1}{2\beta_2^2};$$

$$K_3=\frac{\mathrm{sh}\beta_2\xi_1\mathrm{ch}\beta_2\xi_1+\cos\beta_2\xi_1\mathrm{sh}\beta_2\xi_1}{2\beta_2};\quad K_4=\cos\beta_2\xi_1+\mathrm{ch}\beta_2\xi_1;\quad K'_4=-4\beta_2^4K_1.$$

In this case, the state of an arbitrary cross section ξ_1 of the first segment can be described in the following matrix form:

$$\bar{W}_1(\xi_1)=V_{12}(\xi_1)\bar{W}_{10},$$

where

$$V_{12}(\xi_1)=\begin{pmatrix} K_4(\beta_2\xi_1) & K_3(\beta_2\xi_1) & K_2(\beta_2\xi_1) & K_1(\beta_2\xi_1) \\ -4\beta_2^4K_1(\beta_2\xi_1) & K_4(\beta_2\xi_1) & K_3(\beta_2\xi_1) & K_2(\beta_2\xi_1) \\ -4\beta_2^4K_2(\beta_2\xi_1) & -4\beta_2^4K_1(\beta_2\xi_1) & K_4(\beta_2\xi_1) & K_3(\beta_2\xi_1) \\ -4\beta_2^4K_3(\beta_2\xi_1) & -4\beta_2^4K_2(\beta_2\xi_1) & -4\beta_2^4K_1(\beta_2\xi_1) & K_4(\beta_2\xi_1) \end{pmatrix}. \quad (12)$$

3) if $\tilde{\omega} = \bar{\omega}_0$, then by using the serial integration of the equation

$$W_1^{\mathrm{IV}}=0,$$

we obtain the function

$$W_1=W_{10}+W'_{10}\xi_1+W''_{10}\frac{\xi_1^2}{2}+W'''_{10}\frac{\xi_1^3}{6}$$

and the matrix equation

$$\bar{W}_1(\xi_1)=V_{13}(\xi_1)\bar{W}_{10}, \quad (14)$$

where

$$V_{13}(\xi_1)=\begin{pmatrix} 1 & \xi_1 & \frac{\xi_1^2}{2} & \frac{\xi_1^3}{6} \\ 0 & 1 & \xi_1 & \frac{\xi_1^2}{2} \\ 0 & 0 & 1 & \xi_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Thus, the strain-stress state of the first segment is described by equations (8), (11) and (14) that all together can be expressed in the form

$$\bar{W}_{1j}\left(\xi_1\right) = V_{1j}\left(\xi_1\right)\bar{W}_{10}, \quad j = 1, 2, 3. \tag{15}$$

Transverse vibrations of 2-nd segment $0 \le \xi_2 \le 1 - \nu$

Natural transverse oscillations of this segment can be described by the equation [25–27]

$$\frac{\partial^4 w_2}{\partial \xi_2^4} + 4\alpha^4 \frac{\partial^2 w_2}{\partial \tau^2} = 0. \tag{16}$$

Separating variables by the representation

$$w_2\left(\xi_2, \tau\right) = W_2\left(\xi_2\right)\sin \bar{\omega}\tau, \tag{17}$$

we obtain

$$W_2^{\text{IV}} - \tilde{\omega}^2 W_2 = 0,$$

from where, assuming

$$W_2\left(\xi_2\right) = Ae^{s\xi_2}, \tag{18}$$

we deduce the characteristic equation

$$s^4 - \tilde{\omega}^2 = 0,$$

the roots of which are of the form

$$s_{1,2} = \pm\beta_3; \quad s_{3,4} = \pm i\beta_3; \quad \beta_3 = \sqrt{\tilde{\omega}}. \tag{19}$$

Express the state of an arbitrary cross section ξ_2 of the 2-nd segment by the corresponding vector

$$\bar{W}_2 = \left\{W_2\left(\xi_2\right)\ W_2'\left(\xi_2\right)\ W_2''\left(\xi_2\right)\ W_2'''\left(\xi_2\right)\right\}^T$$

and the matrix equation

$$\bar{W}_2 = V_{11}\left(\beta_3\xi_2\right)\bar{W}_{20}, \tag{20}$$

where $\bar{W}_{20} = \left\{W_{20}\ W_{20}'\ W_{20}''\ W_{20}'''\right\}^T$ is initial parameters vector for the 2-nd segment. Using the conjunction condition between the segments, we obtain

$$\bar{W}_{20} = \bar{W}_2(0) = \bar{W}_1(\nu),$$

since the matrix $v_{11}(0)$ is unit. Then the vector $\bar{W}_{20} = V_{1j}(\nu)\bar{W}_{10}$ $(j = 1, 2, 3)$ and the state vector for both the segments can be expressed via the initial parameters for the first segment

$$\begin{gathered}
\bar{W}_1\left(\xi_1\right) = V_{11}\left(\beta_1\xi_1\right)\bar{W}_{10} \quad \left(\tilde{\omega} > \bar{\omega}_0\right) \\
\bar{W}_1\left(\xi_1\right) = V_{12}\left(\beta_2\xi_1\right)\bar{W}_{10} \quad \left(\tilde{\omega} < \bar{\omega}_0\right) \\
\bar{W}_1\left(\xi_1\right) = V_{13}\left(\xi_1\right)\bar{W}_{10} \quad \left(\tilde{\omega} = \bar{\omega}_0\right) \\
\bar{W}_2\left(\xi_2\right) = V_{11}\left(\beta_3\xi_2\right)V_{1j}(\nu)\bar{W}_{10}.
\end{gathered} \tag{21}$$

Transverse oscillations of a beam with free endpoints

A beam resting on an elastic foundation without restrictions at the endpoints can be described as a proper model of spread footing. In this case, the boundary conditions read:

$$\begin{gathered}
W_{10}'' = W_{10}''' = 0 \\
w_2''\left(1-\nu\right) = w_2'''\left(1-\nu\right) = 0,
\end{gathered} \tag{22}$$

From here it follows that

$$\bar{W}_{10} = \{W_{10}\ W_{10}'\ 0\ 0\}^T$$
$$\bar{w}_2(1-\nu) = \{w_2(1-\nu)\ w_2'(1-\nu)\ 0\ 0\}^T. \quad (23)$$

1. At first, we accept a condition according to which the unknown frequency $\tilde{\omega}$ for a partially supported beam equals to "conventional" frequency $\bar{\omega}_0$. Then, according to (21) and (14), we have

$$\bar{W}_1(\xi_1) = V_{13}(\xi_1)\bar{W}_{10}$$
$$\bar{W}_2(\xi_2) = V_{11}(\xi_2)V_{13}(\nu)\bar{W}_{10}. \quad (24)$$

Let us write the second equation (24) in the expanded form by taking into account (23) and $\xi_2 = 1-\nu$

$$\begin{pmatrix} w_2(1-\nu) \\ w_2'(1-\nu) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} R_4(\beta_3(1-\nu)) & R_3(\beta_3(1-\nu)) & R_2(\beta_3(1-\nu)) & R_1(\beta_3(1-\nu)) \\ \beta_3^4 R_1(\beta_3(1-\nu)) & R_4(\beta_3(1-\nu)) & R_3(\beta_3(1-\nu)) & R_2(\beta_3(1-\nu)) \\ \beta_3^4 R_2(\beta_3(1-\nu)) & \beta_3^4 R_1(\beta_3(1-\nu)) & R_4(\beta_3(1-\nu)) & R_3(\beta_3(1-\nu)) \\ \beta_3^4 R_3(\beta_3(1-\nu)) & \beta_3^4 R_2(\beta_3(1-\nu)) & \beta_3^4 R_1(\beta_3(1-\nu)) & R_4(\beta_3(1-\nu)) \end{pmatrix} \times$$

$$\times \begin{pmatrix} 1 & \nu & \frac{\nu^2}{2} & \frac{\nu^3}{6} \\ 0 & 1 & \nu & \frac{\nu^2}{2} \\ 0 & 0 & 1 & \nu \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} W_{10} \\ W_{10}' \\ 0 \\ 0 \end{pmatrix}.$$

From here, we obtain the homogenous system of equations relatively the unknown initial parameters W_{10} and W_{10}':

$$\begin{cases} R_2(\beta_3(1-\nu))W_{10} + (\nu R_2(\beta_3(1-\nu)) + R_1(\beta_3(1-\nu)))W_{10}' = 0 \\ R_3(\beta_3(1-\nu))W_{10} + (\nu R_3(\beta_3(1-\nu)) + R_2(\beta_3(1-\nu)))W_{10}' = 0. \end{cases}$$

The condition of existing of nonzero roots for this system is the equality to zero the determinant

$$\begin{vmatrix} R_2(\beta_3(1-\nu)) & R_2(\beta_3(1-\nu)) + R_1(\beta_3(1-\nu)) \\ R_3(\beta_3(1-\nu)) & \nu R_3(\beta_3(1-\nu)) + R_2(\beta_3(1-\nu)) \end{vmatrix} = 0.$$

Expanding this determinant, we obtain the frequency equation

$$\mathrm{ch}(\beta_3(1-\nu))\cos(\beta_3(1-\nu)) = 1,$$

the roots of which are [17]

$\beta_{31}(1-\nu) = 0;\ \beta_{32}(1-\nu) = 4{,}73;\ \beta_{33}(1-\nu) = 7{,}853;\ \beta_{34}(1-\nu) = \frac{2n+1}{2}\pi$ at $n > 3$ gives physically impossible results at $\nu = 1$

$$\lim_{\nu\to1}\beta_{32} = \lim_{\nu\to1}\sqrt{\tilde{\omega}} = \lim_{\nu\to1}\frac{4{,}73}{1-\nu} = \infty \text{ and etc.}$$

Consequently, the accepted condition $\tilde{\omega} = \bar{\omega}_0$ is not realized.

2. As is known [25], a free (i.e., without foundation) beam without constrains at its endpoints has two null frequencies corresponding to translational and rotational motion of the beam as a rigid body in addition to

the frequencies of free oscillations that coincide with the frequencies of the beam with clamped endpoints. Consequently, the rigid body motion should be added to the deflections caused by beam's vibrations. Such complex motion is described by the function

$$W = C_1 + C_2\xi.$$

Following to the accepted model of the system «beam-foundation», the existence of small length $\nu \neq 0$ of beam's part interacting with foundation excludes the possibility of beam's motion as a rigid body. At the same time, calculation of the basic first frequency of natural oscillations can be performed according to (10)–(11) that is when the condition $\tilde{\omega} < \bar{\omega}_0,$ holds, beginning from $\omega_1 = 0$ at $\nu = 0$ and $\omega_0 \neq 0.$ Let us accept this condition, that is the unknown frequency $\tilde{\omega}$ is smaller than the «conventional» frequency $\bar{\omega}_0$. Then, according to (21) and (12) we have:

$$\bar{W}_1(\xi_1) = V_{12}(\beta_2\xi_1)\bar{W}_{10}; \quad \bar{W}_2(\xi_2) = V_{11}(\beta_3\xi_2)V_{12}(\beta_2\nu)\bar{W}_{10}. \tag{25}$$

Let us write the second equation (25) using the expanded form for $\xi_2 = 1-\nu$

$$\begin{pmatrix} w_2(1-\nu) \\ w_2'(1-\nu) \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} R_4(\beta_3(1-\nu)) & R_3(\beta_3(1-\nu)) & R_2(\beta_3(1-\nu)) & R_1(\beta_3(1-\nu)) \\ \beta_3^4 R_1(\beta_3(1-\nu)) & R_4(\beta_3(1-\nu)) & R_3(\beta_3(1-\nu)) & R_2(\beta_3(1-\nu)) \\ \beta_3^4 R_2(\beta_3(1-\nu)) & \beta_3^4 R_1(\beta_3(1-\nu)) & R_4(\beta_3(1-\nu)) & R_3(\beta_3(1-\nu)) \\ \beta_3^4 R_3(\beta_3(1-\nu)) & \beta_3^4 R_2(\beta_3(1-\nu)) & \beta_3^4 R_1(\beta_3(1-\nu)) & R_4(\beta_3(1-\nu)) \end{pmatrix} \times$$

$$\times \begin{pmatrix} K_4(\beta_2\nu) & K_3(\beta_2\nu) & K_2(\beta_2\nu) & K_1(\beta_2\nu) \\ -4\beta_2^4 K_1(\beta_2\nu) & K_4(\beta_2\nu) & K_3(\beta_2\nu) & K_2(\beta_2\nu) \\ -4\beta_2^4 K_2(\beta_2\nu) & -4\beta_2^4 K_1(\beta_2\nu) & K_4(\beta_2\nu) & K_3(\beta_2\nu) \\ -4\beta_2^4 K_3(\beta_2\nu) & -4\beta_2^4 K_2(\beta_2\nu) & -4\beta_2^4 K_1(\beta_2\nu) & K_4(\beta_2\nu) \end{pmatrix} \begin{pmatrix} W_{10} \\ W_{10}' \\ 0 \\ 0 \end{pmatrix}. \tag{26}$$

Also, the two linear homogenous equations relatively the unknown initial parameters W_{10} and W_{10}' can be found from the matrix equation (26) as it was done in the Section 1 of the paper:

$$\begin{cases} U_1 W_{10} + \mathrm{U}_2 W_{10}' = 0 \\ U_3 W_{10} + U_4 W_{10}' = 0, \end{cases} \tag{27}$$

where

$$U_1 = \beta_3^4 R_2(X)K_4(Y_1) - 4\beta_2^4\left(\beta_3^4 R_1(X)K_1(Y_1) + R_4(X)K_2(Y_1)) + R_3(X)K_3(Y_1)\right);$$

$$U_2 = \beta_3^4\left(R_2(X)K_4(Y_1) + R_1(X)K_4(Y_1)\right) - 4\beta_2^4\left(R_4(X)K_1(Y_1) + R_3(X)K_2(Y_1)\right);$$

$$U_3 = \beta_3^4\left(R_3(X)K_4(Y_1) - 4\beta_2^4\left(R_2(X)K_1(Y_1) + R_1(X)K_2(Y_1)\right)\right) - 4\beta_2^4 R_1(X)K_3(Y_1);$$

$$U_4 = \beta_3^4\left(R_3(X)K_3(Y_1) + R_2(X)K_4(Y_1) - 4\beta_2^4 R_1(X)K_1(Y_1)\right) - 4\beta_2^4 R_4(X)K_2(Y_1);$$

$$X = \beta_3(1-\nu); \quad Y_1 = \beta_2\nu.$$

Now, obtain the frequency equation by equating the determinant value of the system (27) with zero:

$$U_1U_4 - U_2U_3 = 0. \tag{28}$$

The deflection functions $w_i(\xi_i)$ $(i = 1, 2)$ along with the bending moments $w_i''(\xi_i)$ in arbitrary cross sections are found from the matrix equations (25). For the first segment ($0 \leq \xi_1 \leq \nu$) we obtain

$$w_1(\xi_1) = W_{10}\left(K_4(\beta_2\xi_1) - UK_3(\beta_2\xi_1)\right); \quad w_1''(\xi_1) = W_{10}\left(-K_2(\beta_2\xi_1) - UK_1(\beta_2\xi_1)\right)$$

and the same for the second one ($0 \le \xi_2 < 1 - \nu$):

$$w_2(\xi_2) = W_{10}\sum_{n=1}^{4} R_n(\beta_2\xi_2)P_n;\ \ w_2''(\xi_2) = W_{10}\sum_{n=1}^{4} R_n''(\beta_3\xi_2)P_n,$$

where

$$P_1 = 4\beta_2^4\left(-K_3(Y_1) + UK_2(Y_1)\right);\quad P_2 = 4\beta_2^4\left(-K_2(Y_1) + UK_1(Y_1)\right);$$

$$P_3 = -4\beta_2^4 K_1(Y_1) - UK_4(Y_1);\quad P_4 = K_4(Y_1) - UK_3(Y_1);\quad U = \frac{U_3}{U_4}.$$

The second and higher frequencies and modes of natural vibrations of a beam with free endpoints resting on elastic foundation should be evaluated by assuming that the "conventional" frequency $\bar{\omega}_0$ is smaller than the unknown one $\tilde{\omega}$ in accordance with the variant (6)–(7). It is determined by the requirement that the second frequency (following the first null) of a free beam (i.e., a beam without foundation, when $\bar{\omega}_0 = 0$) equals to 4.73 [25]. Then, according to (21) and (9) we have:

$$\bar{W}_1(\xi_1) = V_{11}(\beta_1\xi_1)\bar{W}_{10};\quad \bar{W}_2(\xi_2) = V_{11}(\beta_3\xi_2)V_{11}(\beta_1\nu)\bar{W}_{10}. \tag{29}$$

From here, we obtain the frequency equation by performing the actions similar to ones in Section 1 and 2

$$Z_1Z_4 - Z_2Z_3 = 0, \tag{30}$$

where

$$Z_1 = b_3^4 R_2(X)R_4(Y_2) + b_1^4\left(b_3^4 R_1(X)R_1(Y_2) + R_4(X)R_2(Y_2) + R_3(X)R_3(Y_2)\right);$$

$$Z_2 = b_3^4\left(R_2(X)R_3(Y_2) + R_1(X)R_4(Y_2)\right) + b_1^4\left(R_4(X)R_1(Y_2) + R_3(X)R_2(Y_2)\right);$$

$$Z_3 = \beta_3^4\left(R_3(X)R_4(Y_2) + \beta_1^4\left(R_2(X)R_1(Y_2) + R_1(X)R_2(Y_2)\right)\right) + \beta_1^4 R_4(X)R_3(Y_2);$$

$$Z_4 = \beta_3^4\left(R_3(X)R_3(Y_2) + R_2(X)R_4(Y_2) + \beta_1^4 R_1(X)R_1(Y_2)\right) + \beta_1^4 R_4(X)R_2(Y_2);$$

$$Y_2 = \beta_1\nu.$$

The deflection functions $w_i(\xi_i)$ $(i = 1, 2)$ and the bending moments $w_i''(\xi_i)$ in arbitrary cross section of the first segment ($0 \le \xi_1 \le \nu$) are found from equation (29) as this was done in Section 1:

$$w_1(\xi_1) = W_{10}\left(R_4(\beta_1\xi_1) - ZR_3(\beta_1\xi_1)\right)$$

$$w_1''(\xi_1) = W_{10}\left(R_2(\beta_1\xi_1) - ZR_1(\beta_1\xi_1)\right)$$

and the same for the second segment ($0 \le \xi_2 \le 1 - \nu$):

$$w_2(\xi_2) = W_{10}\sum_{n=1}^{4} R_n(\beta_3\xi_2)S_n;\ \ w_2''(\xi_2) = W_{10}\sum_{n=1}^{4} R_n''(\beta_3\xi_2)S_n,$$

where

$$S_1 = \beta_1^4\left(R_3(Y_2) - ZR_2(Y_2)\right);\quad S_2 = \beta_1^4\left(R_2(Y_2) - ZR_1(Y_2)\right);$$

$$S_3 = \beta_1^4 R_1(Y_2) - ZR_4(Y_2);\quad S_4 = R_4(Y_2) - ZR_3(Y_2);\quad Z = \frac{Z_3}{Z_4}.$$

3. Results and Discussion

Table 1 shows the values of the first three dimensionless natural frequencies which are obtained from equations (28) and (30) for different combinations of "conventional" frequency values $\bar{\omega}_0$, that characterize generalized stiffness of the system "beam–foundation" and *v* is segment's length after partial destruction of the foundation.

Table 1. The first three frequencies of beam's natural oscillations.

$\bar{\omega}_0$	$\tilde{\omega}_1\left(\tilde{\omega}<\bar{\omega}_0\right)$			$\tilde{\omega}_3\left(\tilde{\omega}>\bar{\omega}_0\right)$			$\tilde{\omega}_3\left(\tilde{\omega}=\bar{\omega}_0\right)$		
	v = 0.25	v = 0.5	v = 0.75	v = 0.25	v = 0.5	v = 0.75	v = 0.25	v = 0.5	v = 0.75
2	0.038	0.268	0.889	22.4	22.42	22.44	61.68	61.685	61.7
6	0.195	1.245	3.309	22.61	22.77	22.94	61.75	61.83	61.89
10	0.412	2.31	4.991	23.08	23.48	23.961	61.86	62.08	62.28
14	0.664	3.25	6.985	23.87	24.53	25.48	62.05	62.47	62.87
18	0.937	4.04	8.174	25.11	25.92	27.46	62.25	63	63.65
22	1.219	4.69	9.078	26.94	27.67	29.83	62.57	63.68	64.61
26	1.498	5.25	9.772	29.27	29.87	32.52	63.04	64.51	65.76
30	1.772	5.76	10.24	32.03	32.59	35.47	63.52	65.5	67.06
34	2.031	6.25	11.15	35.16	35.8	38.6	64	66.63	68.53

Figure 2 shows the dependencies of frequency on parameter $\bar{\omega}_0$ for various lengths v of the supported segment.

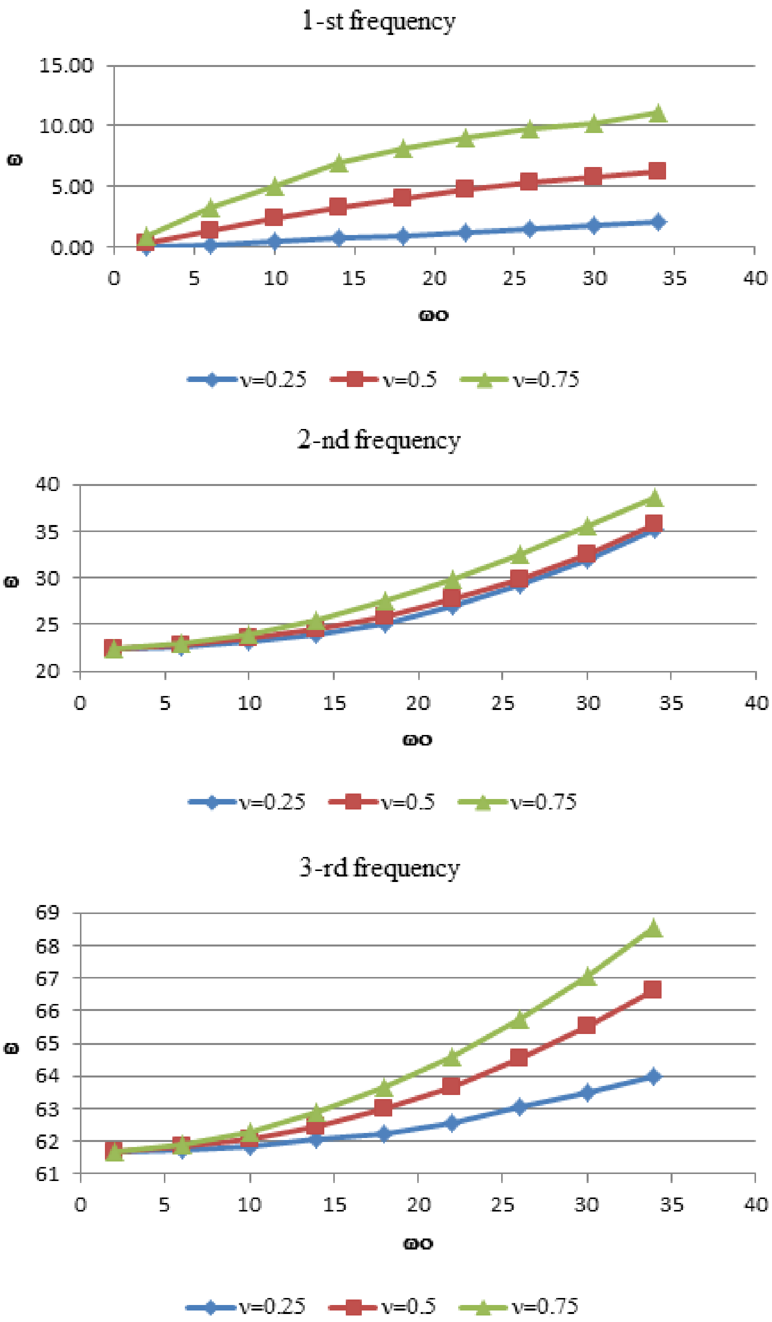


Figure 2. Dependency between frequency of vibrations and generalized stiffness ω_0.

Figure 3 presents modes of natural oscillations corresponding to these frequencies for $\bar{\omega}_0$ = 18 and ν = 0.5.

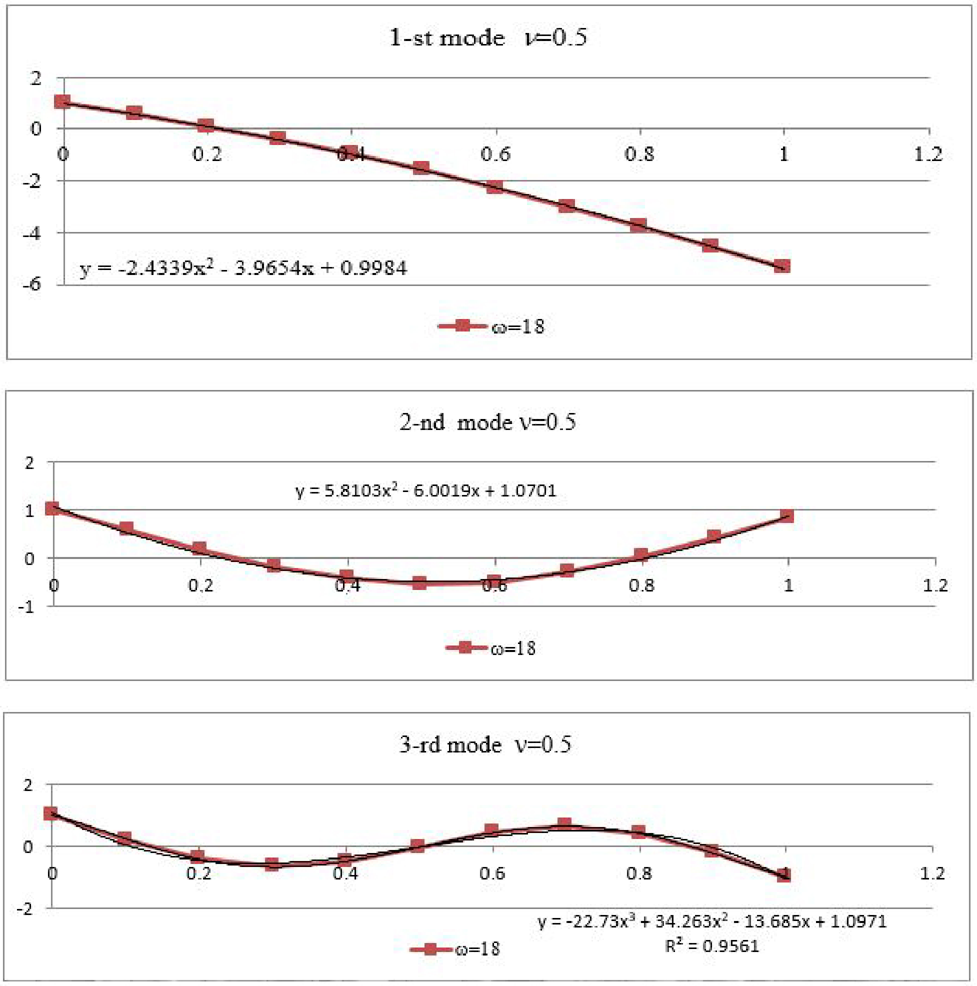


Figure 3. The first three modes of beam's vibrations.

The following equation describes beam's forced oscillations caused by sudden partial damage of a foundation which supports a loaded beam [27]

$$\frac{\partial^4 w_{dyn}}{\partial \xi^4} + 4\alpha^4 \left(w_{dyn} + \frac{\partial^2 w_{dyn}}{\partial \tau^2} \right) = \bar{q}, \tag{31}$$

where $\bar{q} = \dfrac{ql^3}{EI}$ is dimensionless intensity of an evenly distributed load;

$w_{dyn} = w_{dyn}(\xi, \tau)$ is deflection function for an arbitrary cross section ξ ($0 \le \xi \le 1$);

τ is physical time. Let us separate variables in equation (31) using the series

$$w_{dyn} = \sum_{n=1}^{\infty} Q_n(\tau) W_n(\tau), \tag{32}$$

where $W_n = W_n(\xi)$ is eigen function obtained by conjunction of eigen functions $W_{1m}(\xi_1)$ and $W_{2m}(\xi)$; for both the segments;

$Q_n = Q_n(\tau)$ is unknown time function.

We obtain the equations that allow determining the functions $Q_n(\tau)$:

$$\frac{d^2 Q_n}{d\tau^2} + \bar{\omega}_n^2 Q_n = R_n, \tag{33}$$

where

$$R_n = \frac{1}{\bar{\omega}_0^2} \frac{\int_0^1 \bar{q} W_n(\xi) d\xi}{\int_0^1 W_n^2(\xi) d\xi}.$$

The common solution of the equation (31) takes the form [25]

$$w_{dyn} = \sum_{n=1}^{\infty} \left(D_{1n} \cos \bar{\omega}_n \tau + D_{2n} \sin^2 \bar{\omega}_n \tau + \frac{R_n}{\bar{\omega}_n^2} \right) W_n(\xi). \quad (34)$$

The integration constants D_{1n} and D_{2n} can be determined from the initial conditions

$$w_{dyn}(\xi, 0) = w_{st}(\xi), \qquad \left. \frac{\partial w_{dyn}}{\partial \tau} \right|_{\xi, 0} = 0, \quad (35)$$

where $w_{st}(\xi)$ is static deflection of a beam entirely supported by an elastic foundation. This deflection can be determined from the equation [27] taking into account the constraints at the endpoints of the beam:

$$\frac{\partial^4 w_{st}}{\partial \xi^4} + 4\alpha^4 w_{st} = \bar{q}.$$

For a beam simply supported by an elastic foundation of Winkler's type and loaded with evenly distributed load $\bar{q}$ = const, the deflection in the foundation (without flexure) descends versus the depth according to the law

$$w_{st}(\xi) = \frac{\bar{q}}{4\alpha^4}. \quad (36)$$

From 2-nd condition (35), it follows

$$D_{2n} = 0. \quad (37)$$

From 1-st condition (35), we obtain

$$\sum_{n=1}^{\infty} \left(D_{1n} + \frac{R_n}{\bar{\omega}_n^2} \right) w_n(\xi) = w_{st}. \quad (38)$$

Multiplying both the parts of (37) by $W_n(\xi)$ and integrating by ξ from 0 to 1 we obtain

$$D_{1n} = B_n - \frac{R_n}{\omega_0^2}, \quad B_n = \frac{\int_0^1 w_{st} w_n(\xi) d\xi}{\int_0^1 w_n^2(\xi) d\xi}. \quad (39)$$

Substituting (37) and (39) into the series (34) and taking into account the equality

$$1 - \cos \bar{\omega}_m \tau = 2 \sin^2 \frac{\bar{\omega}_n}{2} \tau,$$

we obtain

$$w_{dyn}(\xi, \tau) = \sum_{n=1}^{\infty} \left(B_n \cos \bar{\omega}_n \tau + C_n \sin^2 \frac{\bar{\omega}_n}{2} \tau \right) W_n(\xi), \quad (40)$$

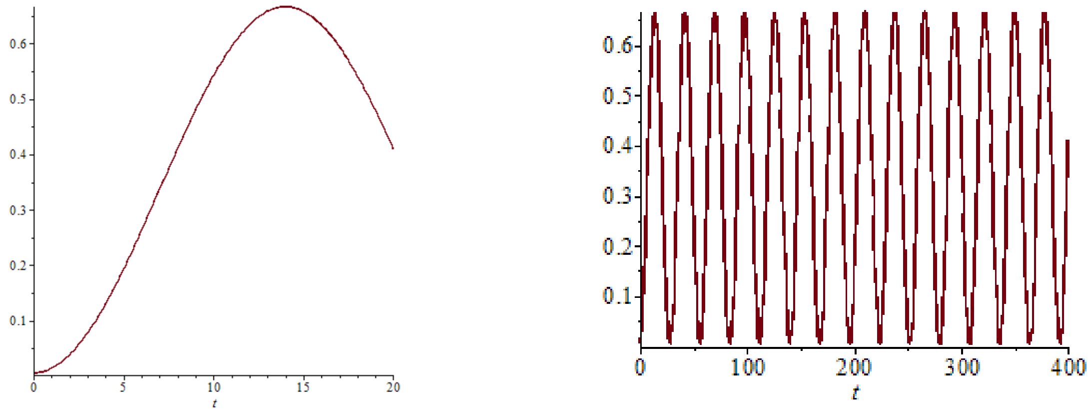
where

$$C_n = \frac{2\overline{q}}{\tilde{\omega}_n^2} \frac{\int_0^1 W_n(\xi)d\xi}{\int_0^1 W_n^2(\xi)d\xi}.$$

Using a similar transformation, we obtain the series for bending moments

$$M_{dyn} = w''_{dyn} = \sum_{n=1}^{\infty}\left(B_n \cos\overline{\omega}_n\tau + C_n \sin^2\frac{\overline{\omega}_n}{2}\tau\right)W''_n(\xi). \quad (41)$$

Figure 4 presents the bending moment behavior in the cross section $\xi = 0.43$ at the beginning of the dynamic process in the beam after sudden foundation destruction under a half of the beam ($\nu = 0.5$) when the generalized stiffness of the system "beam–foundation" is $\overline{\omega}_0 = 18$ $(\alpha = 3)$ (Figure 4 a) and the graph of stationary vibrations at $\tau > 14$ (Figure 4 b). The bending moment reaches its maximum value $M_{\max}^{dyn} = 0.666$.



a) The beginning of bending moment increasing in the cross section $\xi = 0.43$

b) stationary oscillations of bending moment in the same cross section

Figure 4. Bending moment after sudden damage of foundation (ν = 0.5).

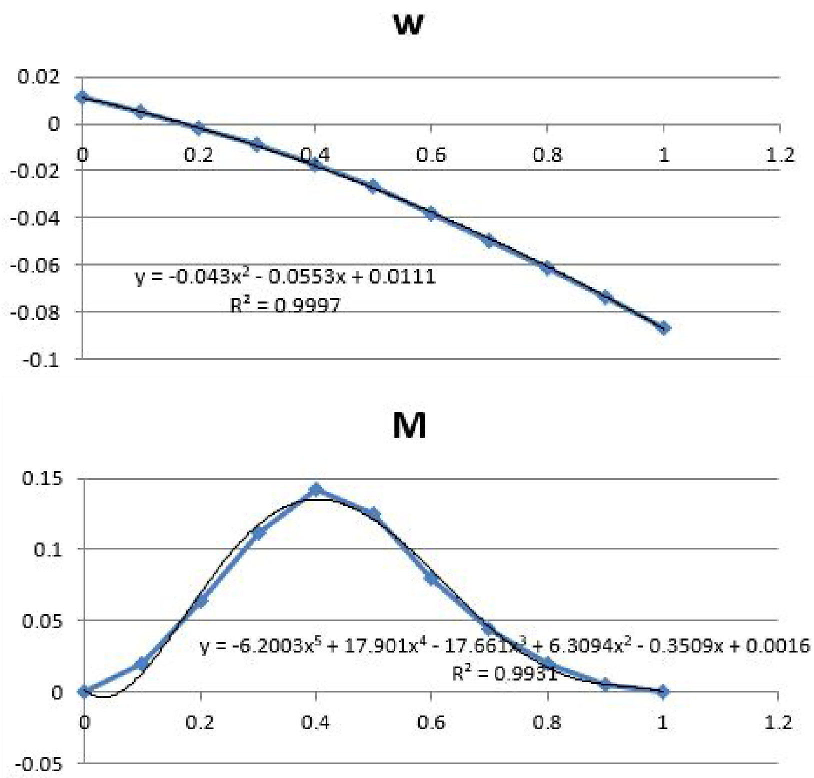


Figure 5. Distribution of deflection and bending moment cased by quasi-static damaging of foundation.

When quasi-static appearing of such a damage, the bending moment reaches it maximum value in the same cross section ξ = 0.43 and is equal to $M_{\max}^{qs}$ = 0.143 (Figure 5) while the external load $\overline{q}$ = 1 is unit. One can see that the effect of sudden damage exhibits as the five-time increase in the internal bending moment.

4. Conclusions

The obtained analytical solution of the problem on determination forces, modes and frequencies of natural and forced flexural vibrations of a beam supported by an elastic foundation can be applied to verification of mathematical models of static-dynamic and quasi-static deforming of complex structural systems «beam – foundation» under accidental impacts caused by sudden destruction a part of foundation. Besides, this analytical solution can be applied to problems on defense of buildings and structures against progressive destruction when, according to the scenario of a special accidental impact, an additional dynamic force due to sudden subsidence of a base in the system «strip footing – structure» plays an important role.

References

1. Pearson, C., Delatte, N. Ronan Point Apartment Tower Collapse and its Effect on Building Codes. Journal of Performance of Constructed Facilities. 2005. 19(2). Pp. 172–177. doi:10.1061/(asce)0887-3828(2005)19:2(172)
2. Travush, V.I., Fedorova, N. V. Survivability of structural systems of buildings with special effects. Magazine of Civil Engineering. 2018. 81(5). Pp. 73–80. doi:10.18720/MCE.81.8
3. Fedorova, N., Savin, S. Structural transformation of reinforced concrete structural system at sudden loss of stability of one of its elements. IOP Conference Series: Materials Science and Engineering. 2018. 365(5). Pp. 052018. doi:10.1088/1757-899X/365/5/052018
4. Li, J., Zhou, H. Energy-based collapse assessment of complex reinforced concrete structures with uncertainties. Procedia Engineering. 2017. 199. Pp. 1246–1251. doi:10.1016/j.proeng.2017.09.262
5. Utkin, V.S. Calculation of the reliability of the earth foundations of buildings and structures according to the deformation criteria with limited information on the soils and loads. Magazine of Civil Engineering. 2016. 61(1). Pp. 4–13. doi:10.5862/MCE.61.1
6. Utkin, V.S., Borisova, O.L. Calculation of the reliability of the slit foundation by the criterion of the bearing capacity of the foundation soil at the stage of operation. Construction of Unique Buildings and Structures. 2017. 57(6). Pp. 7-17. doi: 10.18720/CUBS.57.1
7. Motaghian, S.E., Mofid, M., Alanjari, P. Exact solution to free vibration of beams partially supported by an elastic foundation. Scientia Iranica. 2011. 18(4 A). Pp. 861–866. doi:10.1016/j.scient.2011.07.013
8. Motaghian, S., Mofid, M., Akin, J.E. On the free vibration response of rectangular plates, partially supported on elastic foundation. Applied Mathematical Modelling. 2012. 36(9). Pp. 4473–4482. doi:10.1016/j.apm.2011.11.076
9. Rezaei, E., Dahlberg, T. Dynamic behaviour of an in situ partially supported concrete railway sleeper. Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit. 2011. 225(5). Pp. 501–508. doi:10.1177/2041301710392492
10. Eisenberger, M., Yankelevsky, D.Z.,Clastornik, J. Stability of beams on elastic foundation. Computers and Structures. 1986. 24(1). Pp. 135–139. doi:10.1016/0045-7949(86)90342-1
11. Liu, Y., Shu, D.W. Analytical Solution of the Vibration of Delaminated Bimaterial Beams Fully or Partially Supported by Elastic Foundation. Applied Mechanics and Materials. 2013. 394. Pp. 75–79. doi:10.4028/www.scientific.net/amm.394.75
12. Attar, M., Karrech, A., Regenauer-Lieb, K. Dynamic response of cracked Timoshenko beams on elastic foundations under moving harmonic loads. JVC/Journal of Vibration and Control. 2017. 23(3). Pp. 432–457. doi:10.1177/1077546315580470
13. Tsai, M.H., Huang, T.C. Progressive collapse analysis of an RC building with exterior non-structural walls. Procedia Engineering. 2011. 14. Pp. 377–384. doi:10.1016/j.proeng.2011.07.047
14. Amiri, S., Saffari, H., Mashhadi, J. Assessment of dynamic increase factor for progressive collapse analysis of RC structures. Engineering Failure Analysis. 2018. 84. Pp. 300–310. doi:10.1016/j.engfailanal.2017.11.011
15. Al-Salloum, Y.A., Abbas, H., Almusallam, T.H., Ngo, T., Mendis, P. Progressive collapse analysis of a typical RC high-rise tower. Journal of King Saud University - Engineering Sciences. 2017. 29(4), Pp. 313–320. doi:10.1016/j.jksues.2017.06.005
16. Weng, J., Tan, K.H., Lee, C.K. Adaptive superelement modeling for progressive collapse analysis of reinforced concrete frames. Engineering Structures. 2017. 151. Pp. 136–152. doi:10.1016/j.engstruct.2017.08.024
17. Fedorova, N. V, Savin, S.Y. Ultimate State Evaluating Criteria of RC Structural Systems at Loss of Stability of Bearing Element. IOP Conference Series: Materials Science and Engineering. 2018. 463. Pp. 032072. doi:10.1088/1757-899X/463/3/032072
18. Bažant, Z.P., Verdure, M. Mechanics of Progressive Collapse: Learning from World Trade Center and Building Demolitions. Journal of Engineering Mechanics. 2007. 133(3). Pp. 308–319. doi:10.1061/(ASCE)0733-9399(2007)133:3(308)
19. Khandelwal, K., El-Tawil, S. Pushdown resistance as a measure of robustness in progressive collapse analysis. Engineering Structures. 2011. 33(9). Pp. 2653–2661. doi:10.1016/j.engstruct.2011.05.013
20. Szyniszewski, S., Krauthammer, T. Energy flow in progressive collapse of steel framed buildings. Engineering Structures. 2012. 42. Pp. 142–153. doi:10.1016/j.engstruct.2012.04.014
21. Botez, M., Bredean, L., Ioani, A.M. Improving the accuracy of progressive collapse risk assessment: Efficiency and contribution of supplementary progressive collapse resisting mechanisms. Computers and Structures. 2016. 174. Pp. 54–65. doi:10.1016/j.compstruc.2015.11.002
22. Belostotsky, A.M., Penkovoy, S.B., Scherbina, S.V., Akimov, P.A., Kaytukov, T.B. Correct numerical methods of analysis of structural strength and stability of high-rise panel buildings part 2: Results of modelling. Key Engineering Materials. 2016. 685. Pp. 221–224. doi:10.4028/www.scientific.net/KEM.685.221
23. Travush, V. I., Fedorova, N.V. Survivability parameter calculation for framed structural systems. Russian Journal of Building Construction and Architecture. 2017. 33(1). Pp. 6–14. vestnikvgasu.wmsite.ru/ftpgetfile.php?id=564.
24. Gei, M., Misseroni, D. Experimental investigation of progressive instability and collapse of no-tension brickwork pillars. International Journal of Solids and Structures. 2018. 155. Pp. 81–88. doi:10.1016/j.ijsolstr.2018.07.010

25. Travush, V.I., Gordon, V.A., Kolchunov, V.I., Leontiev, E. V. The response of the system ‘beam - Foundation’ on sudden changes of boundary conditions. IOP Conference Series: Materials Science and Engineering. 2018. 456(1). Pp. 012130. doi:10.1088/1757-899X/456/1/012130
26. Gordon, V.A., Pilipenko, O. V., Trifonov, V.A. The reactions of the «beam – foundation» system to the sudden change of the boundary conditions. MATEC Web of Conferences. 2018. 188. Pp. 03008. doi:10.1051/matecconf/201818803008
27. Travush, V.I., Gordon, V.A., Kolchunov, V.I., Leontiev, Y.V. Dynamic effects in the beam on an elastic foundation caused by the sudden transformation of supporting conditions. International Journal for Computational Civil and Structural Engineering. 2018. 14(4). Pp. 27–41. doi:10.22337/2587-9618-2018-14-4-27-47

Contacts:

Vladimir Travush, +7(495)625-79-67; travush@mail.ru

Vladimir Gordon, +7(486)2419802; gordon@ostu.ru

Vitaly Kolchunov, +7(471)2222461; asiorel@mail.ru

Evgeny Leontiev, +7(495)6259595; e.leontyev@gge.ru

Инженерно-строительный журнал

ISSN 2071-0305

сайт журнала: http://engstroy.spbstu.ru/

DOI: 10.18720/MCE.91.12

Динамическое деформирование балки при внезапном структурном изменении упругого основания

В.И. Травуш[a], В.А. Гордон[b], В.И. Колчунов[c*], Е.В. Леонтьев[d]

[a] Российская академия архитектуры и строительных наук, г. Москва, ул. Россия

[b] Орловский государственный университет имени И.С. Тургенева, г. Орел, Россия

[c] Юго-Западный государственный университет, г. Курск, Россия

[d] Главгосэкспертиза России, г. Москва, Россия

** E–mail: asiorel@mail.ru*

Ключевые слова: система «балка – основание», собственные и вынужденные колебания, формы и частоты, аварийное воздействие, структурная перестройка

Аннотация. Приведена методика аналитического определения усилий, перемещений, форм и частот собственных поперечных колебаний балки на упругом основании, состоящей из двух участков: один опирается на упругое основание Винклера, второй свободен. Уравнения поперечных собственных и вынужденных колебаний балки записаны в безразмерных координатах и решены методом начальных параметров с использованием функций Крылова. При этом вторая и высшие частоты и формы собственных колебаний балки определяются в предположении, что искомая частота больше «условной» частоты, характеризующей обобщенную жесткость системы «балка – основание». Численным анализом показаны зависимости трех первых безразмерных частот собственных колебаний балки от обобщенной жесткости системы «балка – основание» после частичного разрушения основания под балкой. Установлено, что при внезапном разрушении основания под половиной балки при некотором значении обобщенной жесткости системы «балка – основание» эффект внезапной структурной перестройки системы приводит почти к пятикратному увеличению момента.

Литература

1. Pearson C., Delatte N. Ronan Point Apartment Tower Collapse and its Effect on Building Codes // J. Perform. Constr. Facil. 2005. № 2(19). Pp. 172–177.
2. Травуш В.И., Федорова Н.В. Живучесть конструктивных систем сооружений при особых воздействиях // Инженерно-строительный журнал. 2018. № 5(81). С. 73–80. doi:10.18720/MCE.81.8
3. Fedorova N., Savin S. Structural transformation of reinforced concrete structural system at sudden loss of stability of one of its elements // IOP Conf. Ser. Mater. Sci. Eng. 2018. № 5(365). 052018.
4. Li J., Zhou H. Energy-based collapse assessment of complex reinforced concrete structures with uncertainties // Procedia Eng. 2017. № 199. Pp. 1246–1251.
5. Уткин В.С. Расчет надежности грунтовых оснований фундаментов зданий и сооружений по критерию деформации при ограниченной информации о нагрузках и грунтах // Инженерно-строительный журнал. 2016. № 1(61). С. 4–13. doi:10.5862/MCE.61.1
6. Уткин В.С., Борисова О.Л. Расчет надежности щелевого фундамента по критерию несущей способности грунта основания на стадии эксплуатации // Строительство уникальных зданий и сооружений. 2017. № 6(57). С. 7–17. doi: 10.18720/CUBS.57.1
7. Motaghian S.E., Mofid M., Alanjari P. Exact solution to free vibration of beams partially supported by an elastic foundation // Sci. Iran. 2011. № 4 A(18). Pp. 861–866.
8. Motaghian S., Mofid M., Akin J.E. On the free vibration response of rectangular plates, partially supported on elastic foundation // Appl. Math. Model. 2012. № 9(36). Pp. 4473–4482.
9. Rezaei E., Dahlberg T. Dynamic behaviour of an in situ partially supported concrete railway sleeper // Proc. Inst. Mech. Eng. Part F J. Rail Rapid Transit. 2011. № 5(225). Pp. 501–508.
10. Eisenberger M., Yankelevsky D.Z., Clastornik J. Stability of beams on elastic foundation // Comput. Struct. 1986. № 1(24). Pp. 135–139.
11. Liu Y., Shu D.W. Analytical Solution of the Vibration of Delaminated Bimaterial Beams Fully or Partially Supported by Elastic Foundation // Appl. Mech. Mater. 2013. № 394. Pp. 75–79.
12. Attar M., Karrech A., Regenauer-Lieb K. Dynamic response of cracked Timoshenko beams on elastic foundations under moving harmonic loads // JVC/Journal Vib. Control. 2017. № 3(23). Pp. 432–457.
13. Tsai M.H., Huang T.C. Progressive collapse analysis of an RC building with exterior non-structural walls // Procedia Eng. 2011. № 14. Pp. 377–384.

14. Amiri S., Saffari H., Mashhadi J. Assessment of dynamic increase factor for progressive collapse analysis of RC structures // Eng. Fail. Anal. 2018. № 84. Pp. 300–310.
15. Amiri S., Saffari H., Mashhadi J. Assessment of dynamic increase factor for progressive collapse analysis of RC structures // Eng. Fail. Anal. 2018. № 84. Pp. 300–310.
16. Weng J., Tan K.H., Lee C.K. Adaptive superelement modeling for progressive collapse analysis of reinforced concrete frames // Eng. Struct. 2017. № 151. Pp. 136–152.
17. Fedorova N. V, Savin S.Y. Ultimate State Evaluating Criteria of RC Structural Systems at Loss of Stability of Bearing Element // IOP Conf. Ser. Mater. Sci. Eng. 2018. № 463. 032072.
18. Bažant Z.P., Verdure M. Mechanics of Progressive Collapse: Learning from World Trade Center and Building Demolitions // J. Eng. Mech. 2007. № 3(133). Pp. 308–319.
19. Khandelwal K., El-Tawil S. Pushdown resistance as a measure of robustness in progressive collapse analysis // Eng. Struct. 2011. № 9(33). Pp. 2653–2661.
20. Szyniszewski S., Krauthammer T. Energy flow in progressive collapse of steel framed buildings // Eng. Struct. 2012. № 42. Pp. 142–153.
21. Botez M., Bredean L., Ioani A.M. Improving the accuracy of progressive collapse risk assessment: Efficiency and contribution of supplementary progressive collapse resisting mechanisms // Comput. Struct. 2016. № 174. Pp. 54–65.
22. Belostotsky A.M. Correct numerical methods of analysis of structural strength and stability of high-rise panel buildings part 2: Results of modelling // Key Eng. Mater. 2016. № 685. Pp. 221–224.
23. Travush V. I., Fedorova N.V. Survivability parameter calculation for framed structural systems // Russ. J. Build. Constr. Archit. 2017. № 1(33). Pp. 6–14.
24. Gei M., Misseroni D. Experimental investigation of progressive instability and collapse of no-tension brickwork pillars // Int. J. Solids Struct. 2018. № 155. Pp. 81–88.
25. Travush V.I., etc. The response of the system «beam - Foundation» on sudden changes of boundary conditions // IOP Conf. Ser. Mater. Sci. Eng. 2018. № 1(456). 012130.
26. Gordon V.A., Pilipenko O. V., Trifonov V.A. The reactions of the «beam – foundation» system to the sudden change of the boundary conditions // MATEC Web Conf. 2018. № 188. 03008.
27. Travush V.I., Gordon V.A., Kolchunov V.I., Leontiev Y.V. Dynamic effects in the beam on an elastic foundation caused by the sudden transformation of supporting conditions // International Journal for Computational Civil and Structural Engineering. 2018. 14 (4). Pp. 27–41. DOI: 10.22337/2587-9618-2018-14-4-27-47

Контактные данные:

Владимир Ильич Травуш, +7(495)625-79-67; travush@mail.ru
Владимир Александрович Гордон, +7(486)2419802; gordon@ostu.ru
Виталий Иванович Колчунов, +7(471)2222461; asiorel@mail.ru
Евгений Владимирович Леонтьев, +7(495)6259595; e.leontyev@gge.ru